A STUDY ON THE DYNAMIC CHARACTERISTICS OF THE KOREAN YI-DYNASTY BELL TYPE STRUCTURE

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The dynamic characteristics of the Korean Yi-dynasty bell type structure, including the acoustic effects, are analyzed both theoretically and experimentally. The numerical solutions of natural frequencies and mode shape for membrane and flexural behavior are obtained by using the NASTRAN program for the finite element method with plate shell elements of triangular and quadrilateral types. Test bells, A and B types similar to the Kap-Sa bell in Kong-Ju chosen among typical Korean Yi-dynasty bells, are manufactured on different scales to the original bell. To consider the effects of the variation of the structural dimension on the dynamic response, these bells are analyzed with respect to the variation of the thickness of the wall and the bottom rim and the asymmetric Dang-Jwas. The impact method with the Fast Fourier Transform Analyzer is adopted to experimentally assess the dynamic response. Results are in good agreement with the numerical solutions.

Key Words: Bell Type Structure, Dynamic Characteristics, Acoustic Effect, Mode Shape, Finite Element Method.

NOMENCLATURE

G_i	: Nodal points (i=1,2,3,4)
$[K]$	: Stiffness matrix
$[M]$	: Mass matrix
q_i	: Generalized coordinates (i=1,2,3,4,5,6)
u, v, w	: Displacement local coordinate system
X_m	: Material coordinate system
x, y, z	: Local coordinate system
X_G, Y_G, Z_G	: Global coordinate system
α, β, γ	: Angles between local coordinate and diagonal line of quadrilateral elements
$\alpha_1', \alpha_2', \beta_1', \beta_2'$	: Rotation components
θ_m	: Material coordinate rotations

1. INTRODUCTION

In 1884, a study on the vibration of the bells was attempted under the assumption that the low natural frequencies of shells depend mostly on the flexural energy.(Rayleigh, 1884)

Around the late 1920's and the early 1930's, European bells were studied by Jones and Curtiss, and Japanese bells were studied by Aoki and Yamashita. In these studies they primarily investigated the acoustic phenomena by variation of the geometry of the bell. (Jones, 1928, Curtiss, 1935, Yamashita & Aoki, 1932, 1934)

In 1970, an experimental thesis of Komatsuzawa offered data for the design and corrections of the beat frequencies and lingering sound of the Japanese bell. (Komatsuzawa, 1970)

The above-mentioned studies, however, cannot directly by used for the dynamic analysis of the Korean bell because of the following characteristics:

(1) The vibration style of the Korean Yi-dynasty bell is quite different from the European chime bell.

The important sound of the former is the lingering sound, which consists of the fundamental frequencies, but the important sound of the latter is the percussion sound.

(2) Japanese bells are similar to Korean bells with respect to the vibration style, but the configuration of Japanese bells is slightly different from the Korean bells. For instance, Japanese bells have simple spherical heads and cylindrical bodies, and Korean bells have complicated curved and asymmetric heads and non-cylindrical bodies.(Yum, Lee and Kwak, 1980)

Therefore, it is very difficult to analyze detail for the dynamic response of the Korean bell by the analytical method.

Recently, Professor Young Ha Yum and his students studied the vibration analysis of the Korean Yi-dynasty bells, but they employed the approximating theory, in which the bell structure is treated as a simple sphere, cylinder, cone, or any combination of these shapes. (Yum, et al,1980; Yum, Kwak and Chung, 1982; Yum, et al,1981)

To overcome the above-mentioned difficulties and characteristics for the dynamic analysis of the Korean bell, this study uses the Finite Element Method(F.E.M) with plate-shell elements of triangular and quadrilateral types, which have both flexural and extention energies. (NASA, 1980, Bath, et al, 1976)

The computer program used is NASTRAN of the general structural analysis program. (NASA, 1980)

The Impact Method with the Fast Fourier Transformation Analyzer, which is an experimental method, was adopted to verify theoretical solutions.(Yum, et al,1980, Yum, et al,1981, Yum, Kwak and Chung,1982)

Reasonable agreements between the computed and experimental results were achieved.

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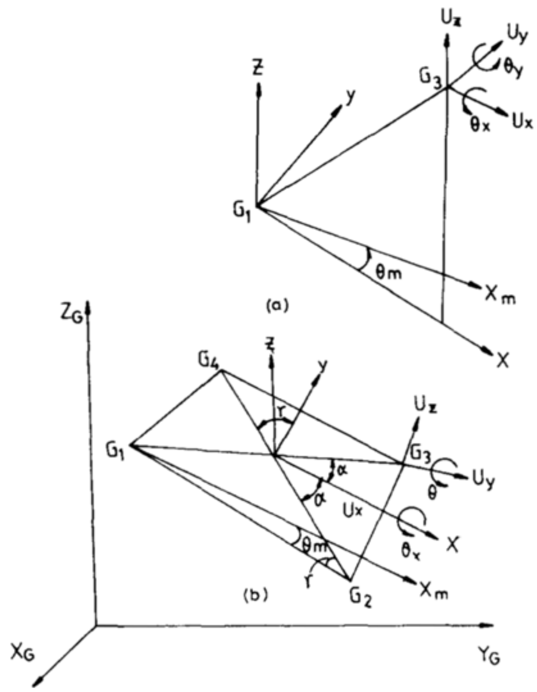

Fig. 1 Coordinate system of plate-shell finite element
(a) Triangular element
(b) Quadrilateral element

2. NUMERICAL METHOD

To analyze the dynamic behavior of the Korean bell with an asymmetric and complicated curved shape, the finite element method, which is the best method to treat numerically, is used.

The element types used are triangular and quadrilateral elements with bending and membrane characteristics.

The coordinate system is shown in Fig. 1.

Triangular plate-shell elements have both membrane stiffness and bending stiffness to be included in transverse shear flexibility.

Anisotropic material properties may be employed in all elements.

The inplane and bending displacements at the corners of the element are represented by the vector

$$\{u\}=[u_1,v_1,w_1,\alpha_1',\beta_1',u_2,v_2,w_2,\alpha_2',\beta_2',u_3,v_3,w_3,\alpha_3',\beta_2']^T \quad (1)$$

where inplane displacements, u and v, are components of displacements parallel to the x and y axes of the element coordinate system, bending deflection, w, is normal to the x-y plane, and α and β are the rotations of the normal to the element which follow the right-hand rule.

Displacement vector $\{u^e\}$ can be expressed by the following equations :

$$u=q_1+q_2x+q_3y \quad (2)$$

$$v=q_4+q_5x+q_6y \quad (3)$$

$$w=r_xx+r_yy+q_1x^2+q_2xy+q_3y^2+q_4x^3+q_5xy^2+q_6y^3 \quad (4)$$

$$\alpha'=q_2x-2q_3y+2q_5xy+3q_6y^2 \quad (5)$$

$$-\beta'=2q_1x+q_2y-3q_4x^2+q_5y^2 \quad (6)$$

$$r_x=\frac{\partial w}{\partial x}+\beta' \quad (7)$$

$$r_y=\frac{\partial w}{\partial y}-\alpha' \quad (8)$$

where r_x and r_y are transverse shear strains and $q_1,q_2,\ldots q_6$ may be regarded as generalized coordinates.

The quadrilateral element uses two sets of overlapping triangular elements.

For each triangle, the x-axis lies along a diagonal so that internal consistency of displacements and rotations of adjacent triangles is assured.

Each triangle has one-half of the bending stiffness assigned to the quadrilateral.

The variational principle may be applied to obtain the following relations.

$$\delta\Pi=0=\delta(U+Ue)-\delta T=\int_A\{u^e\}^T[K]\delta\{u^e\}dA - \int_A\{\ddot{u}^e\}^T[M]\delta\{u^e\}dA \quad (9)$$

The generalized governing equation of the dynamic system becomes :

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{P(t)\} \quad (10)$$

where $[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrices, respectively, and $\{u\}$ and $\{P(t)\}$ are the displacement vector and the applied generalized force vector, respectively

It is the dynamic characteristics of the Korean bell that the sound depends mostly on the first to the third fundamental frequencies which make up the lingering sound.

Thus, the equation of motion becomes.

$$[M]\{\ddot{u}\}+[K]\{u\}=0 \quad (11)$$

To have a non-trivial solution, a general solution of such an equation is written as

$$\{u\}=\{\bar{u}e^{i\omega t}\} \quad (12)$$

where $\bar{u}$ and ω are amplitude vector and natural frequency, respectively. Substituting Eq. (12) into Eq.(11) yields

$$[K]\{\bar{u}\}-\omega^2[M]\{\bar{u}\}=0 \quad (13)$$

or

$$([K]-\lambda[M])\{\bar{u}\}=0 \quad (14)$$

where $\lambda=\omega^2$ are eigen values and $\{\bar{u}\}$ are corresponding eigen vectors.

The method used in this analysis is the Inverse Power Method(IPM). The IPM is particularly effective for problems that are formulated by the displacement approach when only a fraction of all eigenvalues are required, and is also a powerful method for refining the accuracy of eigenvalues and eigen vectors.

The algorithm of the IPM has basically an iteration technique, and it can be explained by the following equation :

$$Let\ \lambda=\lambda_0+\Lambda \quad (15)$$

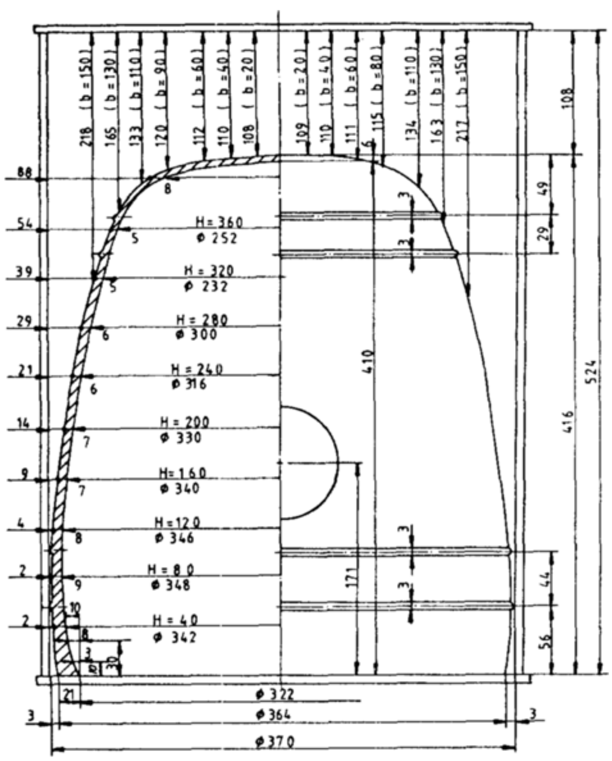

Fig. 2 Model B, Korean bell

where λ_0 is called the shift point.

The iteration algorithm is:

$$(|K|-\lambda_0[M])\{w_n\}=[M]\{\bar{u}_n-1\} \quad (16)$$

$$\{u_n\}=\frac{1}{C_n}\{w_n\} \quad (17)$$

$$\{w_n\}=([K]-\lambda_0[M])^{-1}[M]\{u_n-1\} \quad (18)$$

where C_n is equal to the value of the element of $\{w_n\}$ with the largest absolute value. It is easy to prove that $1/C_n$ converges to Λ, the shifted eigenvalue nearest to the shift point, and that $\{u_n\}$ converges to the corresponding eigenvector.

3. EXPLANATIONS AND ASSUMPTIONS OF THE EXAMPLE

3.1 Numerical Analysis

The analyzed models are model 'A' and model 'B', which are cast from bell bronze(Cu 83.4% and Sn 15.6%) and are scaled down 1/2.5 times from the Kap-Sa bell in Kong-Ju, which is considered the typical Korean Yi-dynasty bell.

To obtain the effect of wall thickness, model 'A'(tave= 10mm) is slightly thicker than model 'B' (tave=6mm).

Model 'B' is shown is Fig. 2.

Assumptions are as follows:

(1) The analyzed models are analized within the elastic limit.

(2) Temperature effects are not considered, for the temperature variations are small since the bells are usually used at atmospheric temperature.

(3) The hanging point of bells is fixed in 6 degrees of freedom, but all other points are free.

(4) The Yong-Doo which is attached at the top of each or the bell is ignored to avoid the analysis difficuties.

The mesh generation is optimized to be close to the original bell shape and to minimize the computing time. The optimizing technique of the mesh generation is the adjustment of the size of the elements, the difference of element number, nodal number, etc. As a result of mesh generation, the total number of elements is 192 with twelve elements in the circumferential direction and 16 elements in the meridional direction.

The mechanical properties of the bell-casting bronze are shown in Table 1.

To investigate the effects of the wall, the rim of the bell, and Dang-Jwas, the analysis procedures are as follows.

(i) The analysis of the two bell modes which have different wall thicknesses.

(ii) The analysis of the effects of the rim thickness.

(iii) The analysis of the asymmetric effects due to the Dang-Jwas. The analyzed results are shown in section 4.

3.2 Experiment

The purpose of the experimental work is to verify the

Table 1 Mechanical properties of the test bell.

Material type	Young's modulus (kg/mm²)	Poisson's ratio	Weight density (kg/mm³)	Tensile strength (kg/mm²)	Impact value (kg−m)	Hardness	
						HRB	HBN
Bronze	9.03×10^3	0.34	8.6×10^{-6}	4.479	11.76	72.8	102

Table 2 Comparison of natural frequencies for model 'B' results of FEM and experiment.

Mode No.		Natural frequencies(Hz)			Mode shape
Flexural	Extension	F.E.M.	Experiment	Deviation	
1		255.8386	225.0000	12(%)	4-0
2		606.1680	637.5000	4.9(%)	6-0
3		911.2571	1002.500	9.1(%)	8-0
	1	1030.215			2-1
4		1098.675	1138.000	3.4(%)	6-1
5		1180.026	1257.000	6.1(%)	10-0
6		1268.436			4-2
7		1327.726	1387.000	4.3(%)	8-1

Fig. 3 Test set up

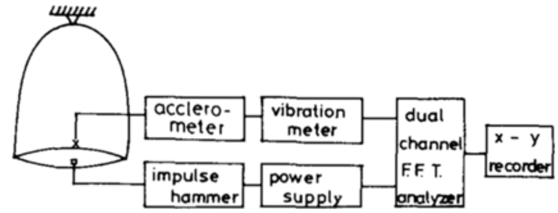

Fig. 4 Instrumentation schematic diagram for natural frequencies and mode test

validity of the computational results described in the previous section.

The experiments are carried out on two bell modes which are cast from bell bronze by the sweep moulding technique and are scaled down 1/2.5 times of the Kap-Sa bell.

The aim of this first experiment is to analyze the free vibration and their corresponding modes of model 'A' and model 'B'. The effects of the rim and Dang-Jwas do not need to be taken into consideration in this experiment. Fig. 3 and Fig. 4 show the experiment set-up for the impulse excitation of the specimen. Instruments used in this experiment are and impulse hammer (PCB Piezotronics), an FFT analyzer (ONO Sokki, CF-500), a micro accelerometer(ONO Sokki NP-501S), an X-Y recorder(ONO Sokki, CX-446), and a charge amplifier(Piezo Electronic Co.). The experimental results will be shown in the next section.

4. RESULTS AND DISCUSSIONS

(1) To verify accuracy, a comparison of the theoretical solutions and experimental results of the natural frequencies and their modes of the model 'B' bell is shown in Table 2. Mode shapes are defined as 4-0, 8-0 and so on where, for instance, '4' means circumferential mode shape and '0' means the longitudinal mode shape.

Figure 5 and Fig. 6 are three dimensional mode shapes and two dimensional mode shapes, respectively, of the model 'B' bell by the theoretical solutions.

Figure 7 shows the variation of the amplitudes with respect to time for natural frequencies of model 'B', and Fig. 8 depicts the two dimensional mode shapes of model 'B' bell by the experimental results.

Comparison of computed natural frequencies with corresponding experimental values shows that, for the model 'B'

Table 3 Comparison of natural frequencies for model 'A' and model 'B', results of F.E.M. and experiment

Flexural Mode No.	Natural frequencies(Hz)						Increment(Hz)			Mode shape
	Model 'A'			Model 'B'						
	FEM	experiment.	deviation.(%)	FEM	experiment.	deviation.(%)	FEM	experiment.	deviation.(%)	
1	392.	350.	10.7	256.	225.	12.0	136.	125.	8.20	4-0
2	917.	925.	0.9	606.	638.	4.9	311.	288.	7.60	6-0
3	1301	1225	5.9	911.	1003	9.1	390.	223.	42.0	8-0

Table 4 Variation of natural frequencies by varying the rim thickness of model 'B'

Flexural mode No.	Original shape (Hz)	Increase thickness (Hz)(+5mm)	Decrease thickness (Hz)(−5mm)	Mode shape
1	255.8386	283.4888	236.0185	4-0
2	606.1680	663.7341	560.4336	6-0

Table 5 Variation of natural frequencies and beat frequencies by the effect of Dang-Jwas.

		Without Dang-Jwas (Hz)	4 Dong-Jwas (180 equi-space)		4 Dang-Jwas (90 equi-space)		Mode shape
			Natural frequencies (Hz)	Beat frequencies (Hz)	Natural frequencies (Hz)	Beat frequencies (Hz)	
1	Low	255.839	255.706	0.173	255.743	0.166	4-0
	High		255.845		255.909		
2	Low	606.168	605.800	1.027	606.458	0.	6-0
	High		606.823		606.458		

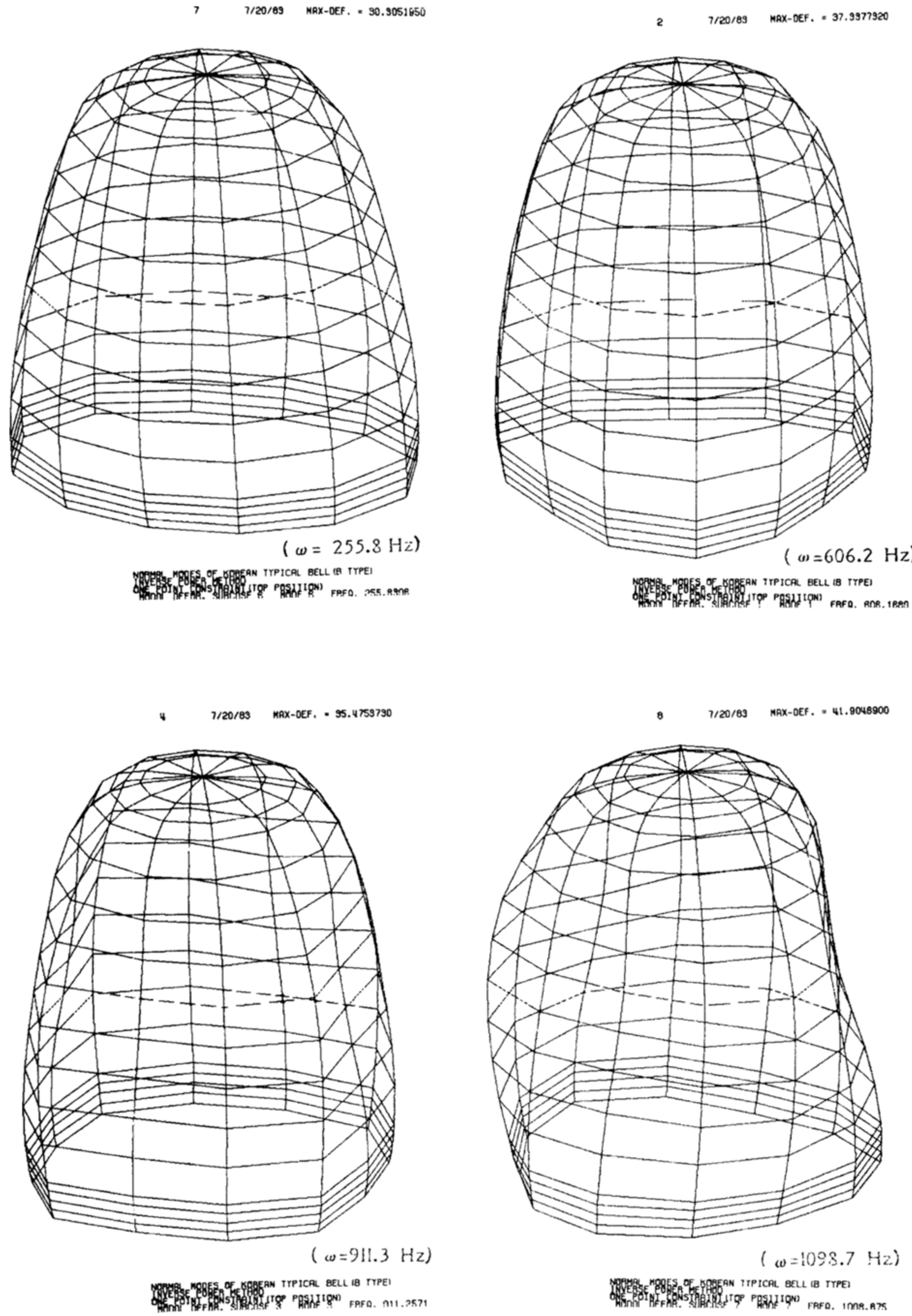

Fig. 5 3 dimensional mode shapes of Model B without asymmetric effect

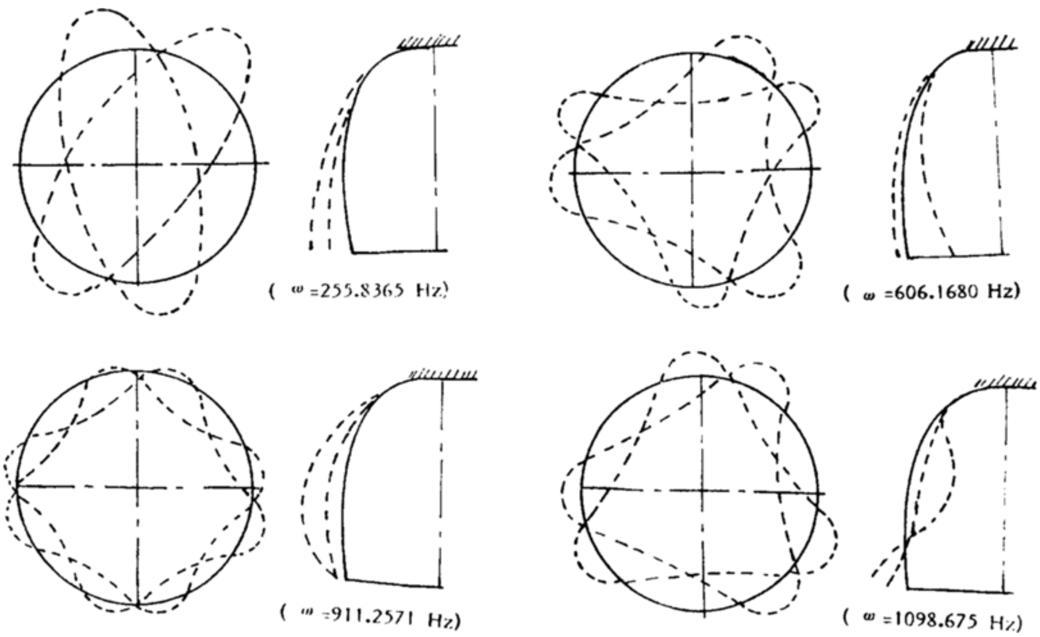

Fig. 6 Circumferential and longitudinal mode shapes Model B without asymmetric effect for F.E.M. results

bell, average deviation is 6.6 percent. The most important reason for this kind of deviation is casting defects which cause the actual values of the bell dimension not to be the same as the designed values.

From the experimental results(Fig. 7), the first and the second flexural mode shapes(4-0 and 6-0) have longer durations of variations than the other mode shapes. Therefore the lingering sound, which is the specific characteristics of the Korean Yi-dynasty bell, depends mostly on these two flexural vibration mode shapes.

(2) Comparisons of the natural frequencies and their corresponding modes between model 'A' and model 'B' by FEM and experiment are shown in Table 3.

For the model 'A' bell, the average deviation is 5.9 percent, similar to that of the model 'B' bell from Table 3, it can be seen that the or a thicker wall not only has higher natural frequencies for corresponding flexural mode shapes, but also a larger size of increments of frequencies with the increment of the mode number.

Therefore, if a designer wants to have a higher frequency bell, it can be done by designing a thicker wall.

(3) Table 4 shows the increase or the decrease of natural frequencies by increasing(+5mm) or decreasing (−5mm) the rim thickness for model 'B' bell by FEM.

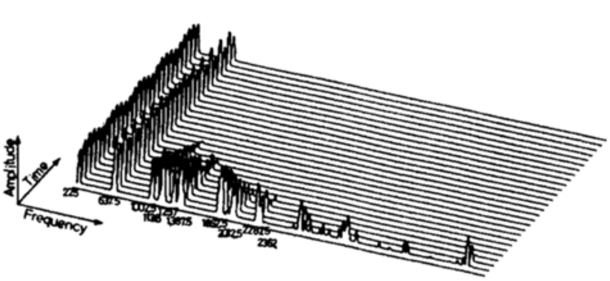

Fig. 7 Variation of the amplitudes vs time for natural frequencies Model B

From the above results, it is found that the thicker rim has the higher frequency and vice versa. Therefore, a designer can easily adjust the frequencies by adjusting the thickness of the rim, even after casting the bell.

(4) Table 5 shows the natural frequencies and beat frequencies produced by the asymmetric effect of the Dang-Jwas which is the striking point of the bell.

From the above results, it is found that Dang-Jwas produce the beat phenomena, with two Dang-Jwas(180 equi-space) having bigger beats than four Dang-Jwas(90 equi-space). Therefore, a designer can adjust the beat by increasing or decreasing the number of Dang-Jwas.

5. CONCLUSION

In this study, the dynamic behaviors of the Korean Yi-dynasty bells were studied and following were concluded:

(1) The vibration style of Korean Yi-dynasty bells depends mostly on the first to third fundamental frequencies of flexural modes, which make up the lingering sound.

(2) The bell with the thicker wall not only has higher natural frequencies, but also has larger sizes of increment of frequencies.

These mode shape characteristics result in a longer life expendency.

(3) The bell with the thicker rim also has higher natural frequencies. Therefore, after manufacturing, natural frequencies can be adjusted easily by reducing the thickness of the rim without large damage.

(4) Asymmetric effects, such as Dang-Jwas, was seen to produce the beat phenomena.

REFERENCES

Aoki, I., Komatsuzawa, A. and Ise, M., 1970, "Effect of Local Loading on the Acoustical Property of Japanse Bell", J.

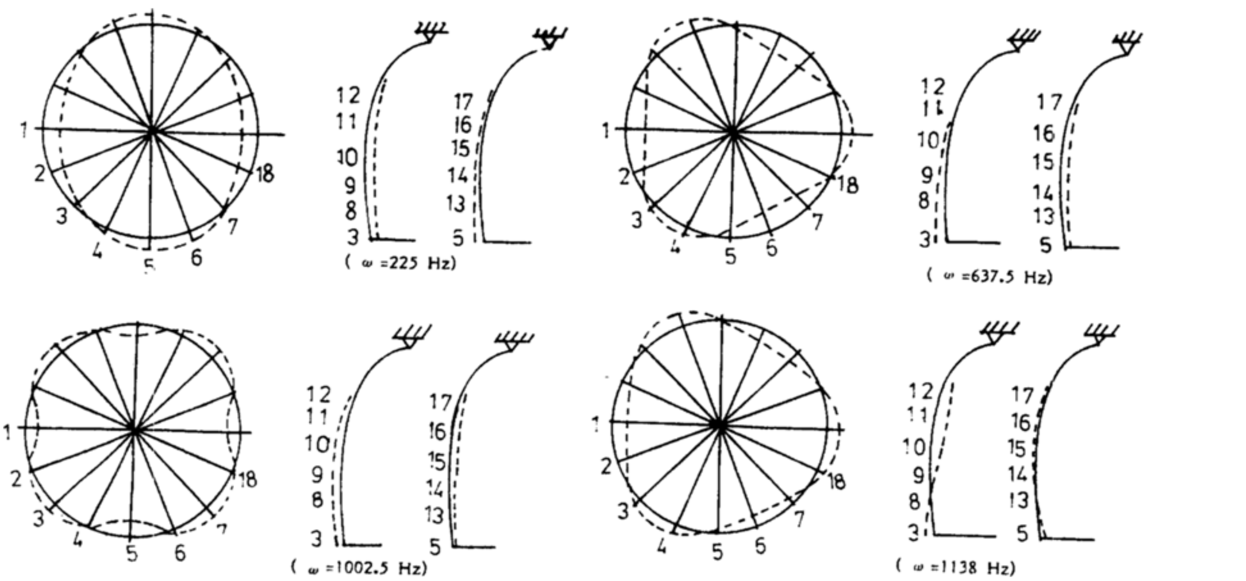

Fig. 8 Circumferential and longitudinal mode shapes of Model B for experimental results

JAS, Vol. 26, No. 12, pp. 323~327.

Arnold, R. N. and Warburton, G. B., 1949, "Flexural Vibrations of the Walls of Thin Cylindrical shell having Freely Supported Ends", Proc.Roy.soc., London, Vol. 17, pp. 238~256.

Bath, K. J. and Wilson, E. L., 1976, "Numerical methods in Finite Element Analysis", Englewood Chiffs,. N.J. Prentice-Hall, pp. 212~230.

Curtiss, A.N. and Gainnini, G.M., 1935, J.A.S.A., Vol. 5, pp. 293, 1933, J.A.S.A., Vol. 24, p. 245

Flügge, W., 1962, "Stress in Shells", Springer-Vertag, Berlin, 2nd 3d., pp. 150~157.

Goleberg, J.E. and Bogdanoff, J.L., 1960, "On the Calculation of the Axisymmetric Modes and Frequencies of Conical Shells", J. Acoust. Soc. Amer., Vol. 5, pp. 130~145.

Jahangir Ansari, 1983, "Dynalic Reponse Analysis of Shells of Revolution due to External Forces using F.E.M.", S.N.U. Ph.D. Thesis.

Jones, A.T., 1928, Phys. Rev., Vol. 31, pp. 1092, 1920, Phys. Rev., Vol. 16, pp. 247~252.

Jones, R.E. and Strome, D.R., 1966, "Direct Stiffness Method Analysis of Shells of Revolution Utilizing Curved Element", AIAA J.,Vol. 16, pp. 390~399.

Kalins, A., 1964, "Free Vibration of Rotationally Symmetric Shell", J.Acoust. Soc. Amer., Vol. 9, pp. 270~283.

Leissa, A.W., 1978, "Vibration of Shell", NASA SP-288.

Love, A.E.H., 1888, "The small Free Vibrations and Deformation of a Thin Elastic Shell", Phill Trans. Royal Soc. London, England(A), Vol. 179, pp. 491~546.

MSC/NASTRAN Primer, 1982, Static & Normal Modes Analysis.

NASTRAN Theoretical Manual, 1980, NASA Sp-221, Section 5.3

Novozhilov, V.V., 1964, "The Theory of Thin Elastic Shells", P.Noordhoff Lts, Chap. 8, pp. 260~351.

Novozhilov, V.V., 1953, "Foundations of Nonlinear Theory of Elasticity", Graylock Press, pp. 47~53.

Oden, L., 1972, "Finite Element of Nonlinear Continua", McGraw-Hill, pp. 5~20.

Percy, J. H., Pian, T.H.H., Klein, S. and Navaratna, R., 1965, "Application of Matrix Displacement Method to Linear Elastic Analysis of Shells of Revolutions", AIAA J., Vol. 8, pp. 231~240.

Rayleigh, J.W.S., 1884, "The Theory of Sound", Dover Pub., London, Vol. 1, pp. 126~132.

Sen, S.K. and Gould, P.L., 1974, "Free Vibration of Shells of Revolution Using F.E.M", J. Eng. Mech., ASCE, EM2, Vol. 3, pp. 261~273.

Timoshenko, S. and Woinowsky-Krieger, 1959, "Theory of Plates and Shells", McGraw-Hill, New York, Chap. 6 pp. 185~189.

Wilkison, J.H., 1965, "The Algebraic Eigenvalue Problem" London Oxford Press, pp. 146~153.

Yamashita, K. and Aoak, L., 1932, "On the Frequencies of the Sound Emitted by Japanese Hanging Bells", Memoirs of the Coll of Sci. Kyoto Univ., Series A, Vol. 15, pp. 572~583.

Yamashita, K. and Aoki, L., 1934, "The Effect of the Length on the Frequencies of Sound Emitted by a Circular Cylindrical with a Hemispherical Cap", Memoirs of the Coll of Sci. Kyoto Univ., Vol. 17, pp. 381~392.

Yum, Y.H., Lee, J.M. and Kwak, J.K., 1980, "A Study on the Vibration characteristics of Bell Structure", Korean Bell Study Report, Bum Jong, No. 3, pp. 31~38.

Yum, Y.H., Lee, J.M. and Kwak, J.K., "A Study on the natural Frequencies and Stress Analysis of a Cylindrical Shell", Korean Bell Study Report, Bum Jong, No. 3, pp. 19~30.

Yum, Y.H., Kwak, J.K. and Chung, S.C., 1982, "A Study on the Natural Frequencies of Sound Emitted by Thin Conical Shells", KSME Trans., Vol. 6, No. 4, pp. 353~360.

Yum, Y.H. and Kim, S.H., 1981, "Study on the Bell Structure", Korean Bell Study Report, Bum Jong, No. 4, pp. 55~68.